4: Entanglement

Saturday, March 13, 2021 3:05 PM

<u>def</u>: A state $|\psi\rangle$ is <u>product</u> if $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

<u>def</u>: A state $|\psi\rangle$ is <u>entangled</u> if $\nexists\ |\psi_1\rangle, |\psi_2\rangle$ such that $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

Q: • Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

Is $|\beta_{00}\rangle$ entangled?

(Try proof by contradiction!)

Q: • Let $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ Entangled?

Assume for contradiction not entangled:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \Longrightarrow \begin{matrix} a \text{ or } d = 0 \\ b \text{ or } c = 0 \end{matrix}$$

$\Downarrow$

$ac = 0$ or $bd = 0$

$\Downarrow$

But $ac = bd = 1/\sqrt{2}$, a contradiction

$\Downarrow$

entangled

Entangled qubits allow 2 physically separated parties to achieve correlated random outcomes that are impossible to create with <u>classical</u> randomness (i.e. coin flips)